Modeling the Stock Market under Uncertain Volatility and Returns

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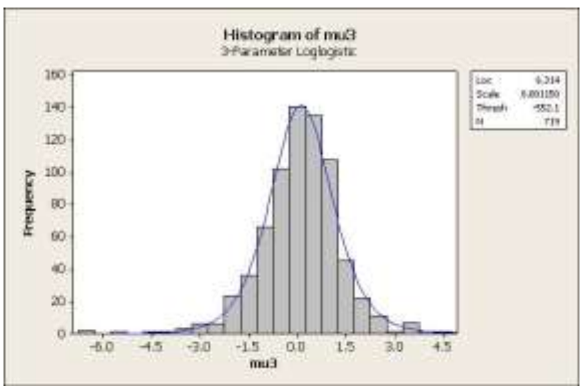
Abstract:
The celebrated Black-Scholes pricing model is used to price options in the market today. This model assumes that at time, t, the price of a stock, S_t, can be expressed by the stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where μ is the return of the stock, and σ is volatility, and W_t is the Brownian Motion random process. Brownian Motion is a family of random variables, $(Wt)_{t\in[0,T]}$, first introduced in 1900 by L. Bachelier to model the stock market. An improvement upon that model is the stochastic volatility model which assumes that μ and σ are uncertain and dependent upon both t and S_t.

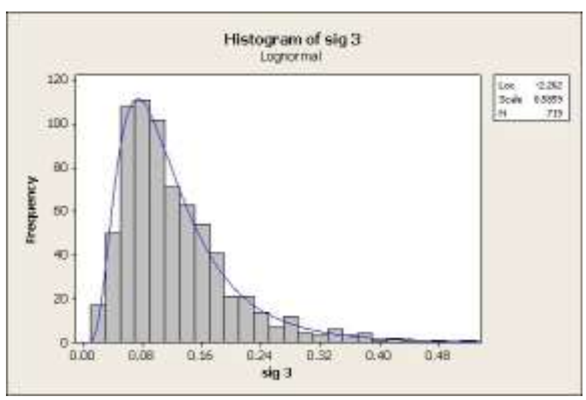
Determining the behavior of the joint distribution of the marginals, μ and σ, is an essential problem in market analysis. This project introduces a new method for modeling the distributions of the marginals. After testing a sample of 25 stocks, including the S&P 500, over a period of 14 years from 1992-2006, it could be concluded that the method was successful on 23 out of 25 of the stocks.



Returns for S&P 500 14-Year Disjoint Weekly

The first step in discovering the model was to calculate the marginals based on a reasonable data grouping. The chosen method of data analysis in this project was a disjoint weekly model. This procedure was chosen because it can be assumed that there is structure that exists within a given week (Monday-Friday) in the volatility and returns on a stock. The method verified this assumption when further analysis proved that the fitting on 4 and 5 day disjoint weeks had even greater structure, implying that consistencies exist in the returns and volatility of a stock during a week.

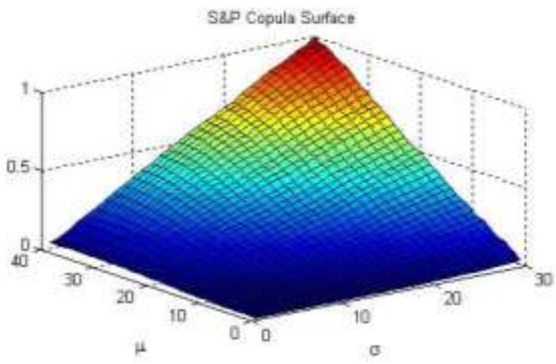
This analysis allowed the marginals to be efficiently modeled by a 3-parameter loglogistic distribution and lognormal distribution for μ and σ, respectively. Both the Anderson Darling statistics (a measurement of the goodness of fit) and the parameters of the distributions confirmed the selection of the method of analysis and the chosen distributions. All of the AD values were statistically acceptable and the parameters were consistently in specific ranges across all of the tested stocks.



Volatility for S&P 500 14-Year Disjoint Weekly

This new approach identified both μ and σ, which means that it pinpointed the family of distributions of the returns for each fixed volatility and the family of distributions of the volatility for each fixed return. The two stocks not able to be modeled under this new method were determined to be anomalous for reasons specifically relating to the stocks themselves.

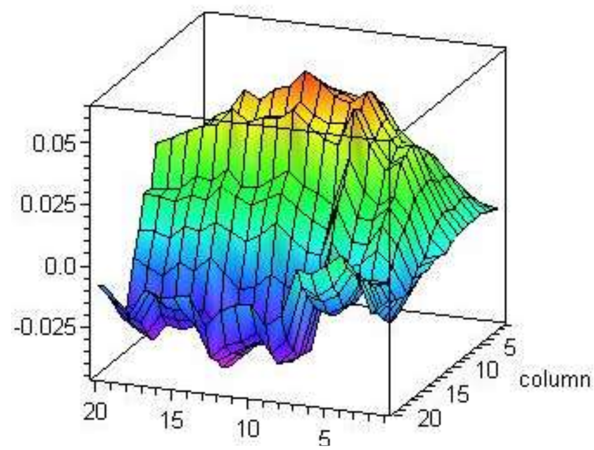
Once identifying the distributions of the marginals, μ and σ, it is important to understand the dependency between the volatility and the returns. The copula method is an appropriate tool to solve this problem. In this method the joint distribution of (μ, σ) is transformed into a new function, called copula, which should be thought of as a dependence encoder. In principle, since the knowledge of the marginals is already built into the construction of the copula, it should be easier to analyze than the joint distribution of μ and σ.



Copula Surface for S&P 500

Unfinished copula work suggests that dependency exists between the returns and the volatility of a stock. This was established by analyzing the copula against its upper and lower Frechet bounds. The dependence exhibited is not extreme. In order to find the exact dependency the copula needs to be analyzed against the independent copula. This analysis confirms that dependency exists between the marginals. Early work indicates that the structure of the dependency may be able to be modeled with a 3-parameter function.

The understanding of both the behavior of the joint distribution and the dependency between the marginals could ultimately be used to model future ranges of stock prices. Applications of this goal would be to price numerous exotic options as well as portfolio management. Current literature suggests that the best existing pricing models today have about 4 parameters, while this model can be expected to have about 8 parameters. This would mean that this new approach could be a more accurate model than any existing today.



Copula of S&P 500 less Independent Copula

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Biography:
Alexandra Agroskin is graduating May 2007 with a B.S. in Applied Mathematics. She will continue her studies at Lehigh next year completing two Masters in Statistics and Analytical Finance. Her interest in economic development will be incorporated into her research next year.

Eric Weaver is graduating May 2007 with a B.A. in Mathematics and Economics and a minor in Engineering. His research interests include the economics of renewable energy and stock market modeling. Next year he will be continuing his studies as a graduate student in the College of Business and Economics seeking a M.S. in Economics.